Non-linear contributions to interactions in climate networks:
sources, relevance, nonstationarity

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Context: Studying global climate structure
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    - dimensionality reduction (poster XY400, Wed 15.30)
    - feature & change detection (poster XY399, Wed 15.30)
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Characterizing dependence

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Pearson’s correlation \( \rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y} \)
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Mutual information:

\[
I(X; Y) = \sum_{y \in Y} \sum_{x \in X} p(x, y) \log \left( \frac{p(x, y)}{p(x)p(y)} \right)
\]
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- Sources/form/origin
- Relevance for higher-order analysis
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Data and methods

Data: NCEP/NCAR reanalysis dataset
  - surface air temperatures
  - monthly data (years 1948 - 2007; 720 timepoints)
  - global grid $73 \times 144$ points (2.5 deg $\times$ 2.5 deg sampling)
  - yearly cycle removed (anomalies)

Methods: interaction/dependence quantification
  - nonlinear: mutual information (pdf estimated using equiprobable binning; $N=8$)
  - linear: Pearson's correlation
  - mutual information on linear surrogate data
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Results: Existence

Controling for method bias:

Statistical testing against 100 surrogates: 15% links above 95th percentile
Results: Existence

Controlling for method bias:
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Controlling for method bias:
Localization of nonlinear contributions
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- Mean MI of a node
- Mean extraMI of a node
- Mean extraMI of a node (relative to meanMI)
Localization of nonlinear contributions
introduce conservative preprocessing: month-wise variance equalization

Statistical testing against surrogates: 8% links above 95th percentile
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Form/origin II

Temperature anomalies:

After additional normalization of variance:
What about remaining ‘non-linearities’?
More examples
Form/origin III

Temperature anomalies:

After additional normalization of variance:
Form/origin III

Temperature anomalies:

After additional detrending:
Conclusion

**existence:** deviations from linear dependences (non-linearities) confirmed
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Questions

▶ What if linear and nonlinear measures disagree?
▶ What about genuine non-linearities?

Thank you for your attention!

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Relevance for graph topology
Donges et al., 2009: nonlinearity key for global topology
Other datasets: ERA